

Observational Constraints of New Variable Modified Chaplygin Gas Model

Jhumpa Bhadra^{1*} and Ujjal Debnath^{2†}

¹*Department of Mathematics,
Heritage Institute of Technology,
Anandapur, Kolkata-700 107, India.*

²*Department of Mathematics,
Bengal Engineering and Science University,
Shibpur, Howrah-711 103, India.*

(Dated: November 13, 2012)

Assuming the flat FRW universe in Einstein's gravity filled with New Variable Modified Chaplygin gas (NVMCG) dark energy and dark matter having negligible pressure. In this research work we analyze the viability on the basis of recent observation. Hubble parameter H is expressed in terms of the observable parameters H_0 , Ω_m^0 and the model parameters A_0 , B_0 , C_0 , m , n , α and the red shift parameter z . Here we find a best fitted parameter range of A_0 , B_0 keeping $0 \leq \alpha \leq 1$ and using Stern data set (12 points) by minimizing the χ^2 test at 66%, 90% and 99% confidence levels. Next we do the joint analysis with BAO and CMB observations. Again evaluating the distance modulus $\mu(z)$ vs redshift (z) curve obtained in the model NVMCG with dark matter with the best fitted value of the parameters and comparing with that derived from the union2 compilation data.

PACS numbers: 98.80.-k, 98.80.Es

I. INTRODUCTION

Modern observations like high redshift survey of SNe Ia [1–4], CMBR [5–9], WMAP [10–14] illustrate the accelerating phase of the universe. Accelerating phase phenomenon of the universe is not understood by the standard Big Bang model of cosmology giving a time like singularity in the past. Cosmological constant Λ with the equation of state $p = -\rho$ is the simplest candidate in Einstein's gravity which describe the present accelerating phase of the universe. The aspects of inflation and the cosmological constant are not understood well. There are various candidates to play the role of the dark energy having negative pressure and describe the accenting view of the present observation. The mystifying fluid namely dark energy is understood to dominate the 70% of the Universe which violets the strong energy condition and 30% dark matter (cold dark matters plus baryons). Various effective candidate of dark energy namely Chaplygin gas with equation of state $p = -\frac{B}{\rho}$, $B > 0$ introduced by [15]. Furthermore it has been generalized to the form $p = -\frac{B}{\rho^\alpha}$, $0 \leq \alpha \leq 1$ [16] and modified to $p = A\rho - \frac{B}{\rho^\alpha}$ [17, 18]. The another candidate of dark energy was introduced by Chakraborty et al [19], known as *New Variable modified Chaplygin Gas* (NVMCG) which follows the equation $p = A(a) - \frac{B(a)}{\rho^\alpha}$, $0 \leq \alpha \leq 1$, a being the scale factor which gives interesting physical significance.

For flat universe having the energy densities for dust like matter and dark energies we need to know the value of critical energy density and $H(z)$ at high accuracy [20]. The MCG best fits with the 3 year WMAP and the SDSS data with the choice of parameters $A = 0.085$ and $\alpha = 1.724$ [21] which are improved constraints than the previous ones $-0.35 < A < 0.025$ [22]. Here we have assumed the new variable modified Chaplygin gas model in flat FRW cosmology. The joint data analysis of stern data set with BAO and CMB have been analyzed for this model. The distance modulus vs redshift has been examined for our model via redshift-magnitude observational data from Supernova type Ia (Union 2).

* bhadra.jhumpa@gmail.com

† ujjaldebnath@yahoo.com , ujjal@iucaa.ernet.in

A. Basic Equations for Einstein Gravity

The Friedmann-Robertson-Walker(FRW) metric is considered as

$$ds^2 = -c^2 dt^2 + a^2(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right] \quad (1)$$

where $a(t)$ is the scale factor of the universe and r, θ, ϕ are the dimensionless comoving co-ordinates, k be the curvature parameter of space-time metric and takes the values $k = 0, +1, -1$ for flat, closed, open universe respectively.

We consider a spatially flat universe ($k = 0$) with dark energy and dark matter (non-interacting). Thus the Einstein equations becomes (choosing $8\pi G = c = 1$)

$$H^2 = \frac{1}{3}\rho \quad (2)$$

$$\dot{H} = -\frac{1}{2}(p + \rho) \quad (3)$$

where $\rho = \rho_{DE} + \rho_m$ the total energy density of the universe and $p = p_{DE} + p_m$ the total pressure. ρ_{DE}, p_{DE} are the energy density and pressure for dark energy respectively and ρ_m, p_m that for dark matter. For non-interacting fluid conservation equations become

$$\dot{\rho}_{DE} + 3H(\rho_{DE} + p_{DE}) = 0 \quad (4)$$

and

$$\dot{\rho}_m + 3H(\rho_m + p_m) = 0 \quad (5)$$

We have assumed that the universe is filled with New variable modified Chaplygin Gas (NVMCG) as dark energy whose EoS is [19]

$$p = A(a)\rho_{DE} - \frac{B(a)}{\rho_{DE}^\alpha} \quad \text{with } 0 \leq \alpha \leq 1 \quad (6)$$

where $A(a)$, and $B(a)$ are function of the scale factor a . In particular, choose $A(a) = A_0 a^{-n}$ and $B(a) = B_0 a^{-m}$ with A_0, B_0, m, n are positive constants. For $n = m = 0$, this model reduces to modified Chaplygin Gas and for $n = 0$, the model reduces to the variable modified Chaplygin gas model.

Expression for the energy density for NVMCG model is obtained from (4) as [19]

$$\rho = a^{-3} \exp \left(\frac{3A_0 a^{-n}}{n} \right) \left[C_0 + \frac{B_0}{A_0} \left(\frac{3A_0(1+\alpha)}{n} \right)^{\frac{3(1+\alpha)+n-m}{n}} \Gamma \left(\frac{m-3(1+\alpha)}{n}, \frac{3A_0(1+\alpha)}{n} a^{-n} \right) \right]^{\frac{1}{1+\alpha}} \quad (7)$$

where C_0 is an integration constant and $\Gamma(s, t)$ is the upper incomplete gamma function.

For dark matter, the EoS is $p_m = 0$ and so $\rho_m = \rho_m^0 a^{-3}$, $\rho_m^0 = 3\Omega_m^0 H_0$. The Hubble parameter (H) in terms of redshift parameter (z) can be expressed as (from eq. (2))

$$3H^2 = (1+z)^3 \left[3H_0 \Omega_m^0 + \exp \left(\frac{3A_0 \left(\frac{1}{1+z} \right)^{-n}}{n} \right) \left\{ C_0 + \frac{B_0}{A_0} \left(\frac{3A_0(1+\alpha)}{n} \right)^{\frac{3(1+\alpha)+n-m}{n}} \right\} \right]$$

$$\times \Gamma \left(\frac{m - 3(1 + \alpha)}{n}, \frac{3A_0(1 + \alpha)}{n} \left(\frac{1}{1+z} \right)^{-n} \right) \left\{ \frac{1}{1+\alpha} \right\}^{1/(1+\alpha)} \quad (8)$$

Subsequently, we investigate the bound on the model parameter by observational data fitting. The parameters are determined by $H(z)$ - z (Stern), BAO and CMB data analysis [24–28] using χ^2 minimization technique from Hubble-redshift data set.

II. OBSERVATIONAL DATA ANALYSIS $H(z)$ - z (STERN), BAO AND CMB DATA AS A CONSTRAINING TOOL

From the above expression we can write, the Hubble parameter $H(z)$ which can be put in the form as

$$H^2(A_0, B_0, \alpha, m, n, C_0, z) = H_0^2 E^2(A_0, B_0, \alpha, m, n, C_0, z) \quad (9)$$

where,

$$E(A_0, B_0, \alpha, m, n, C_0, z) = \frac{(1+z)^{3/2}}{H_0 \sqrt{3}} \left[3H_0 \Omega_m^0 + \exp \left(\frac{3A_0 \left(\frac{1}{1+z} \right)^{-n}}{n} \right) \times \right. \\ \left. \left\{ C_0 + \frac{B_0}{A_0} \left(\frac{3A_0(1 + \alpha)}{n} \right)^{\frac{3(1+\alpha)+n-m}{n}} \Gamma \left(\frac{m - 3(1 + \alpha)}{n}, \frac{3A_0(1 + \alpha)}{n} \left(\frac{1}{1+z} \right)^{-n} \right) \right\}^{1/(1+\alpha)} \right]^{1/2} \quad (10)$$

Now $E(A_0, B_0, \alpha, m, n, C_0, z)$ contains six unknown parameters A_0, B_0, C_0, m, n and α . Now we will fixing two parameters and by observational data set the relation between the other two parameters will obtain and find the bounds of the parameters.

z	Data	$H(z)$	σ
0.00	73	± 8.0	
0.10	69	± 12.0	
0.17	83	± 8.0	
0.27	77	± 14.0	
0.40	95	± 17.4	
0.48	90	± 60.0	
0.88	97	± 40.4	
0.90	117	± 23.0	
1.30	168	± 17.4	
1.43	177	± 18.2	
1.53	140	± 14.0	
1.75	202	± 40.4	

Table 1: The Hubble parameter $H(z)$ and the standard error $\sigma(z)$ for different values of redshift z .

A. Analysis of $H(z)$ - z (Stern) data set

For given $\alpha, m, n, C_0, z, A_0$ and B_0 can be best fitted by minimizing χ_{H-z}^2 given by

$$\chi_{H-z}^2(A_0, B_0, \alpha, m, n, C_0, z) = \sum \frac{(H(A_0, B_0, \alpha, m, n, C_0, z) - H_{obs}(z))^2}{\sigma_z^2} \quad (11)$$

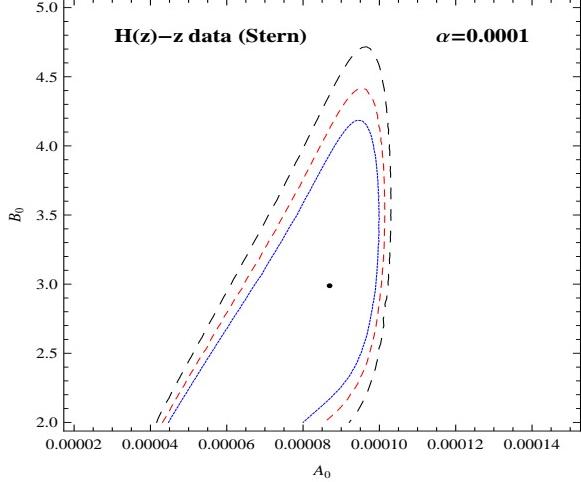


Fig.1

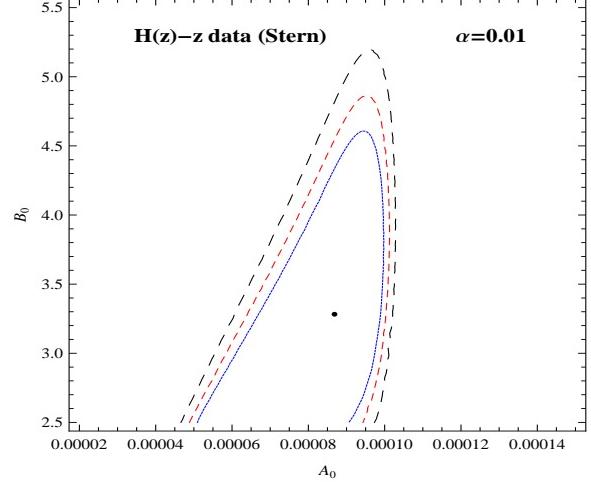


Fig.2

Figs. 1 and 2 show the variation of A_0 with B_0 for $\alpha = 0.0001$ and $\alpha = 0.01$ respectively for different confidence levels. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted for the $H(z)-z$ (Stern) analysis.

where H_{obs} is the observed Hubble parameter at redshift z and σ_z is the error associated with that particular observation (see table 1) and H represents the theoretical values of Hubble parameter calculated for our model. Here we use the observed value of Hubble parameter at different redshifts (twelve data points) listed in observed Hubble data by Stern et al [23] to analyze our model. We consider the present value of Hubble parameter $H_0 = 72 \pm 8 Kms^{-1}Mpc^{-1}$ and a fixed prior distribution. By fixing the model parameter $\alpha \in [0, 1]$, m , n and C_0 , we determine the range of other two parameters A_0 and B_0 by minimizing (11). The probability distribution function in terms of the parameters A_0 , B_0 , C_0 , m , n and α is given by

$$L = \int e^{-\frac{1}{2}\chi^2_{H-z}} P(H_0) dH_0 \quad (12)$$

where $P(H_0)$ is the prior distribution function for H_0 . Now our best fit analysis with Stern observational data support the theoretical range of the parameters. In figures 1 and 2, we plot the graphs for different confidence levels 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours for $\alpha = 0.0001$ and 0.01 respectively and by fixing the other parameters. The best fit values of A_0 , B_0 and the minimum values of χ^2 are tabulated in Table 2.

α	A_0	B_0	χ^2_{min}
0.0001	0.0000869	2.988	32.164
0.01	0.0000868	3.282	31.925

Table 2: $H(z)-z$ (Stern): The best fit values of A_0 , B_0 and the minimum values of χ^2 for $m = 7$, $n = 13$, $C_0 = 0.1$ and for different values of α .

B. Analysis of $H(z)-z$ with BAO Peak Parameter

In this section we use the method of joint analysis, the Baryon Acoustic Oscillation (BAO) peak parameter proposed by Eisenstein et al [29]. Acoustic peaks occurred because cosmological perturbations excite sound waves in initial relativistic plasma in the early epoch of the Universe. Sloan Digital Sky Survey (SDSS) survey is one of the first redshift survey (46748 luminous red galaxies spectroscopic sample)

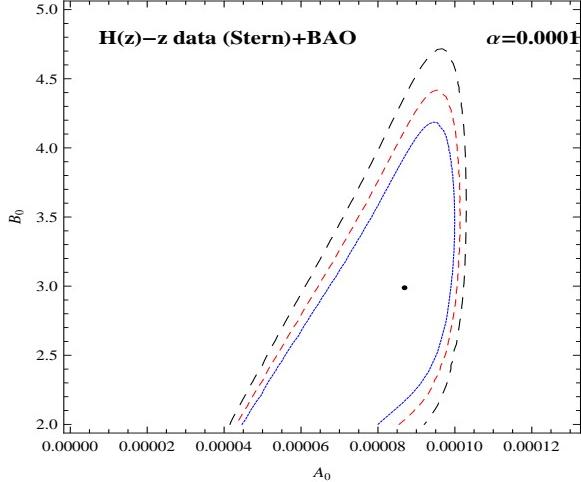


Fig.3

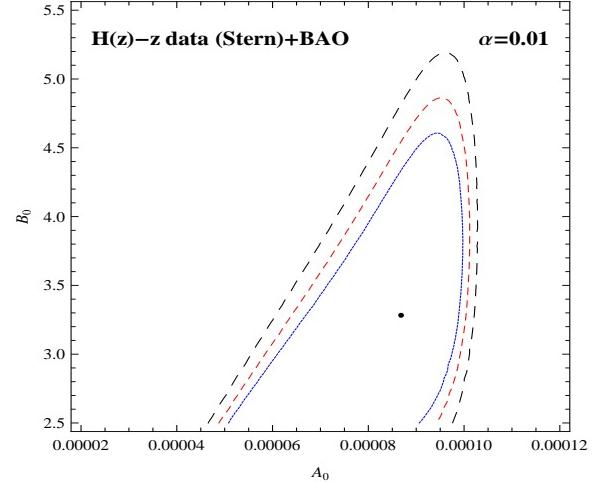


Fig.4

Figs. 3 and 4 show the variation of A_0 with B_0 for $\alpha = 0.0001$ and $\alpha = 0.01$ respectively for different confidence levels. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted for the $H(z)$ - z (Stern)+ BAO joint analysis.

by which the BAO signal has been directly detected at a scale ~ 100 MPc. The corresponding comoving scale of the sound horizon shell is about 150 Mpc in radius. We shall investigate the two parameters A_0 and B_0 for our model using the BAO peak joint analysis for low redshift (with range $0 < z < 0.35$) using standard χ^2 distribution. The BAO peak parameter may be defined by

$$\mathcal{A} = \frac{\sqrt{\Omega_m}}{E(z_1)^{1/3}} \left(\frac{\int_0^{z_1} \frac{dz}{E(z)}}{z_1} \right)^{2/3} \quad (13)$$

where

$$\Omega_m = \Omega_m^0 (1 + z_1)^3 E(z_1)^{-2} \quad (14)$$

Here, $E(z)$ is the normalized Hubble parameter (i.e., $E(z) \equiv E(A_0, B_0, \alpha, m, n, C_0, z)$) and $z_1 = 0.35$ is the typical redshift of the SDSS data sample. This quantity can be used even for more general models which do not present a large contribution of dark energy at early times [30]. Now the χ^2 function for the BAO measurement can be written as in the following form

$$\chi_{BAO}^2 = \frac{(\mathcal{A} - 0.469)^2}{0.017^2} \quad (15)$$

where the value of the parameter \mathcal{A} for the flat model ($k = 0$) of the FRW universe is obtained by $\mathcal{A} = 0.469 \pm 0.017$ using SDSS data set [29] from luminous red galaxies survey. Now the total joint data analysis (Stern+BAO) for the χ^2 function defined by

$$\chi_{Tot}^2 = \chi_{H-z}^2 + \chi_{BAO}^2 \quad (16)$$

Now our best fit analysis with Stern+BAO observational data support the theoretical range of the parameters. In figures 3 and 4, we plot the graphs for different confidence levels 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours for $\alpha = 0.0001$ and 0.01 respectively and by fixing the other parameters. The best fit values of A_0 , B_0 and the minimum values of χ^2 are tabulated in Table 3.

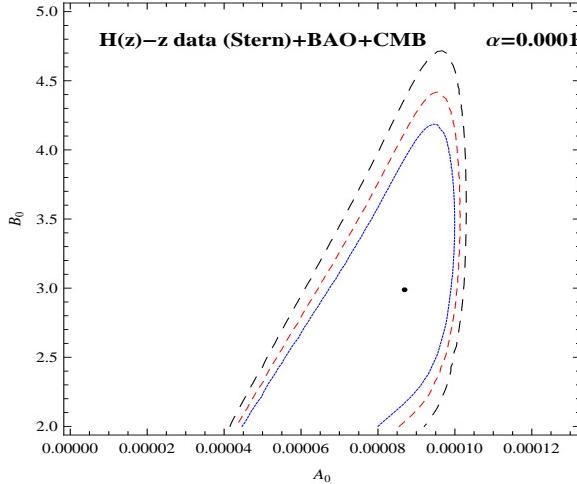


Fig.5

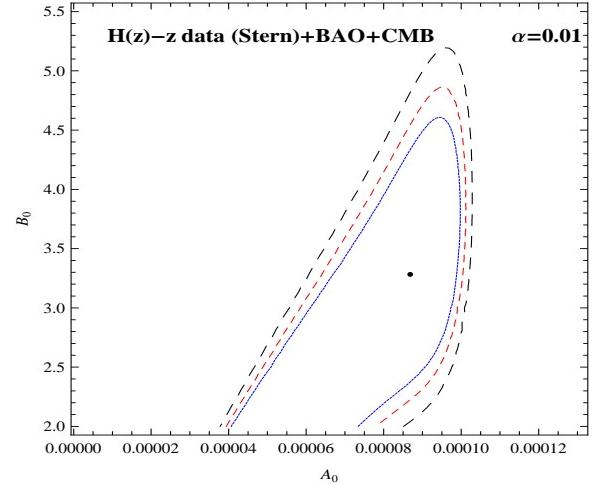


Fig.6

Figs. 5 and 6 show the variation of A_0 with B_0 for $\alpha = 0.0001$ and $\alpha = 0.01$ respectively for different confidence levels. The 66% (solid, blue), 90% (dashed, red) and 99% (dashed, black) contours are plotted for the $H(z)$ - z (Stern) + BAO+ CMB joint analysis.

α	A_0	B_0	χ^2_{min}
0.0001	0.0000868	2.989	32.165
0.01	0.0000867	3.283	31.924

Table 3: $H(z)$ - z (Stern) + BAO: The best fit values of A_0 , B_0 and the minimum values of χ^2 for $m = 7$, $n = 13$, $C_0 = 0.1$ and for different values of α .

C. Analysis with $H(z)$ - z , BAO Peak Parameter and CMB Shift Parameter

Another dynamical parameter that is used in recent cosmological tests is the CMB (Cosmic Microwave Background) shift parameter which is the useful quantity to characterize the position of the CMB power spectrum first peak. The CMB power spectrum first peak is the shift parameter which is given by [31, 32]

$$\mathcal{R} = \sqrt{\Omega_m} \int_0^{z_2} \frac{dz'}{H(z')/H_0} \quad (17)$$

where z_2 is the value of z at the surface of last scattering. WMAP data gives $\mathcal{R} = 1.726 \pm 0.018$ at $z = 1091.3$. For CMB measurement χ^2 function can be defined as

$$\chi^2_{CMB} = \frac{(\mathcal{R} - 1.726)^2}{(0.018)^2} \quad (18)$$

and the total joint data analysis (Stern+BAO+CMB) for the χ^2 function defined by

$$\chi^2_{Tot} = \chi^2_{H-z} + \chi^2_{BAO} + \chi^2_{CMB} \quad (19)$$

Now our best fit analysis with Stern + BAO + CMB observational data support the theoretical range of the parameters. In figures 5 and 6, we plot the graphs for different confidence levels 66% (solid, blue),

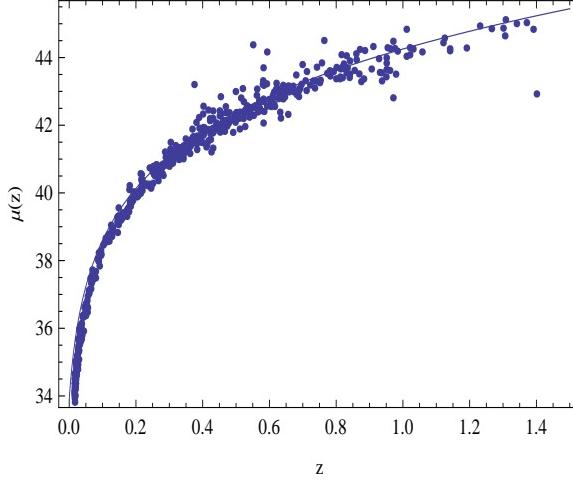


Fig.7

Fig.7 shows the variation of distance modulus $\mu(z)$ vs redshift z for our model (solid line) and the Union2 sample (dotted points).

90% (dashed, red) and 99% (dashed, black) contours for $\alpha = 0.0001$ and 0.01 respectively and by fixing the other parameters. The best fit values of A_0 , B_0 and the minimum values of χ^2 are tabulated in Table 4.

α	A_0	B_0	χ^2_{min}
0.0001	0.0000869	2.987	32.165
0.01	0.0000867	3.283	31.926

Table 4: $H(z)$ - z (Stern) + BAO + CMB: The best fit values of A_0 , B_0 and the minimum values of χ^2 for $m = 7$, $n = 13$, $C_0 = 0.1$ and for different values of α .

III. REDSHIFT-MAGNITUDE OBSERVATIONS FROM SUPERNOVAE TYPE Ia

Recent observation of high redshift survey Supernovae Type Ia indicates that the universe undergoing an accelerating phase and gives an evidence of existence of dark energy. Since 1995, two teams of High-z Supernova Search and the Supernova Cosmology Project have discovered several type Ia supernovas at the high redshifts [1–3]. The observations directly measure the distance modulus of a Supernovae and its redshift z [33]. Recent observational data, including SNe Ia which consists of the gold sample which has a 157 supernova [34] and the another set is a combined data set of a 192 supernova [33]. Here we consider 557 data points and belongs to the Union2 sample [35] which are considered in the next.

The luminosity distance $d_L(z)$ and distance modulus $\mu(z)$ for Supernovas are calculated by

$$d_L(z) = (1+z)H_0 \int_0^z \frac{dz'}{H(z')} \quad (20)$$

and

$$\mu(z) = 5 \log_{10} \left[\frac{d_L(z)/H_0}{1 Mpc} \right] + 25 \quad (21)$$

The best fit of distance modulus as a function $\mu(z)$ of redshift z for our theoretical model and the Supernova Type Ia Union2 sample are drawn in figure 7 for our best fit values of A_0 , B_0 with the other

chosen parameters. From the curves, we see that the theoretical NVMCG model is in agreement with the union2 sample data.

IV. DISCUSSION

We proposed here the FRW universe filled with dark matter (perfect fluid with negligible pressure) along with new variable Modified Chaplygin gas (NVMCG) which is one of the candidates of dark energy. We present the Hubble parameter H in terms of the observable parameters Ω_m^0 , H_0 with the redshift z and the other parameters like A_0 , B_0 , C_0 , α , m and n . We have chosen the observed values of $\Omega_m^0 = 0.28$ and $H_0 = 72 \text{ Kms}^{-1} \text{ Mpc}^{-1}$. From Stern data set (12 points), we have obtained the bounds of the arbitrary parameters A_0 and B_0 (Table 2) by minimizing the χ^2 test and by fixing the other parameters $m = 7$, $n = 13$, $\alpha = 0.1$, 0.0001 and $C_0 = 0.1$. In this way, we may found the bounds of any two parameters by fixing the remaining parameters. Next due to joint analysis of BAO and CMB observations, we have also obtained the best fit values and the bounds of the parameters (A_0, B_0) (Table 3 and 4) by fixing some other parameters $m = 7$, $n = 13$, $C_0 = 0.1$ and $\alpha = 0.01$, 0.0001 . The best-fit values and bounds of the parameters are obtained by 66%, 90% and 99% confidence levels are shown in figures 1-6 for Stern, Stern+BAO and Stern+BAO+CMB analysis. The distance modulus $\mu(z)$ against redshift z has been drawn in figure 7 for our theoretical model of the NVMCG for the best fit values of the parameters and the observed SNe Ia Union2 data sample. Here we show that our predicted theoretical NVMCG model permitted the observational data sets. The observations do in fact severely constrain the nature of allowed composition of matter-energy by constraining the range of the values of the parameters for a physically viable NVMCG model.

Acknowledgement:

The authors are thankful to IUCAA, Pune, India for warm hospitality where part of the work was carried out. Also UD is thankful to CSIR, Govt. of India for providing research project grant (No. 03(1206)/12/EMR-II).

- [1] Perlmutter, S. J. et al, 1998, Nature 391, 51.
- [2] Perlmutter, S.J. et al, 1999, *Astrophys. J.* 517, 565.
- [3] Riess, A. G. et al., 1998, *Astron. J.* 116, 1009.
- [4] Tonry, J. L. et al., 2003, *ApJ*, 594, 1.
- [5] Melchiorri A. et al.,2000,*Astrophysics J.Lett.*,536,L63.
- [6] Lange A.E. et al.,2001,*Phys Rev. D*, 63,042001.
- [7] Jaffe A.H. et al.,2001,*Phys Rev. Lett.*,86,3475.
- [8] Netterfield C.B. et al.,2002,*Astrophysics,J.*,571,604.
- [9] Halverson N.W. et al.,2002,*Astrophysics. J.*,568,38.
- [10] Briddle S. et al.,2003,*Science*,299,1532.
- [11] Bennet C. et al.,2003,[*astro-ph/0302207*].
- [12] Hinshaw G. et al.,2003,[*astro-ph/0302217*].
- [13] Kogut A. et al.,2003,[*astro-ph/0302213*].
- [14] Spergel D.N. et al.,2003,[*astro-ph/0302209*].
- [15] A. Kamenshchik et al, *Phys. Lett. B* **511** 265 (2001).
- [16] Gorini, V., Kamenshchik, A. and Moschella, U., 2003, *Phys. Rev. D* 67, 063509.
- [17] H. Benaoum, *hep-th/0205140*.
- [18] Debnath, U., Banerjee, A. and Chakraborty, S., 2004, *Class. Quantum Grav.* 21, 5609.
- [19] W. Chakraborty and U. Debnath, *Gravitation and Cosmology*, **16** 223 (2010).
- [20] Choudhury, T. R. and Padmanabhan, T., 2007, *Astron. Astrophys.* 429, 807.
- [21] Lu, J. et al, 2008, *Phys. Lett. B* 662, 87.
- [22] Dao-Jun, L. and Xin-Zhou, L., 2005, *Chin. Phys. Lett.*, 22, 1600.
- [23] Stern, D. et al, 2010, *JCAP* 1002, 008.
- [24] Wu, P. and Yu, H., 2007, *Phys. Lett. B* 644, 16.

- [25] Thakur, P., Ghose, S. and Paul, B. C., 2009, Mon. Not. R. Astron. Soc. 397, 1935.
- [26] Paul, B. C., Ghose, S. and Thakur, P., arXiv:1101.1360v1 [astro-ph.CO].
- [27] Paul, B. C., Thakur, P. and Ghose, S., arXiv:1004.4256v1 [astro-ph.CO].
- [28] Ghose, S., Thakur, P. and Paul, B. C., arXiv:1105.3303v1 [astro-ph.CO].
- [29] Eisenstein, D. J. et al, 2005, Astrophys. J. 633, 560.
- [30] M. Doran, S. Stern and E. Thommes, JCAP 0704, 015 (2007).
- [31] O. Elgaroy and T. Multamaki, Astron. Astrophys. 471, 65E (2007).
- [32] G. Efstathiou and J. R. Bond, Mon. Not. R. Ast. Soc. 304, 75 (1999).
- [33] A. G. Riess et al., Astrophys. J. 659, 98 (2007); Astrophys. J. 659, 98 (2007).
- [34] A. G. Riess et al., Astrophys. J. 607, 665 (2004).
- [35] Kowalaski et al, 2008, Astrophys. J. 686, 749.